

# Could wormholes form in dark matter galactic halos?

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**Abstract** We estimate expression for velocity as a function of the radial coordinate  $r$  by using polynomial interpolation based on the experimental data of rotational velocities at distant outer regions of galaxies. The interpolation technique has been used to estimate fifth degree polynomial followed by cubic spline interpolation. This rotational velocity is used to find the geometry of galactic halo regions within the framework of Einstein's general relativity. In this letter we have analyzed features of galactic halo regions based on two possible choices for the dark matter density profile, viz. Navarro, Frenk & White (NFW) type (Navarro, Frenk & White 1996) and Universal Rotation Curve (URC) (Castignani et al. 2012). It is argued that spacetime of the galactic halo possesses some of the characteristics needed to support traversable wormholes.

**Keywords** General Relativity; rotation curves of galaxies; galactic wormholes

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## 1 Introduction

During last several decades wormhole has been attracting attention to the scientific community a lot after since publication of the seminal work by Morris & Thorne (1982). In this paper they argued the possibility of the existence of traversable wormholes permitting to travel through space and time. Actually, a wormhole does act role for a passage/tunnel in spacetime which is supposed to connect the widely separated regions of our universe or different universes in the multiverse model. According to Morris & Thorne (1982), the normal matter is unable to hold a wormhole open rather the matter is responsible for sustaining a traversable wormhole is exotic in nature which violates the standard null energy condition.

By drawing our attention to the galactic level where one can notice a kind of peculiar phenomena known as the flat rotation curves in galaxies. Now, the ordinary luminous matters in space are composed mainly of neutral hydrogen clouds. But this bizarre galactic rotation curves can not be explained by the standard model. Therefore, to explain the rotation curves in the outer regions of galaxies it has been supposed that galaxies and even clusters of galaxies must contain some non-luminous matter. This kind of exotic stuff, now known as dark matter, does not emit electromagnetic waves nor interact with normal matter. It has arguably proved by the scientists that dark matter can explain properly the so called flat rotation curves.

In the previous some studies in connection to flat rotation curves (Rahaman et al. 2014a,b) we have used *ad hoc* functional forms of rotation velocity. However, by using the experimental data of the rotational velocities at different distance of outer regions of galaxies, we have estimated fifth degree polynomial that yields the expression for the velocity as a function of the radial coordinate  $r$  which is almost of the same nature of the

experimental feature. Thus in this paper, we study the exact observational results and confirm the existence of wormhole in all the galaxies containing the dark matter. These dark matters actually play the role of fuel for developing wormhole-like geometry. This study is a combination of Einstein's theory and experimental result, therefore, reasonably more physical than the previous studies (Rahaman et al. 2014a,b).

## 2 The basic equations and their solutions

To connect the dark matter and galactic rotation curves with wormholes one needs to introduce the metric for a static spherically symmetric spacetime. Following Morris & Thorne (1982) we consider here the line element for the galactic halo region in connection to wormhole spacetime as

$$ds^2 = -e^{2f(r)} dt^2 + \left(1 - \frac{b(r)}{r}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (1)$$

where  $c = G = 1$  in the geometrized unit.

Till now we have some idea regarding the density profile, however, other properties of dark matter yet to be explored. Therefore, it is justified to assume that dark matter is characterized by the general anisotropic energy-momentum tensor (Bozorgnia et al. 2013)

$$T_\nu^\mu = (\rho + p_t)u^\mu u_\nu - p_t g_\nu^\mu + (p_r - p_t)\eta^\mu \eta_\nu, \quad (2)$$

where  $u^\mu u_\mu = -\eta^\mu \eta_\mu = -1$ . Here  $p_t$  and  $p_r$  are transverse and radial pressures, respectively.

Using Navarro-Frenk-White (NFW) (Navarro, Frenk & White 1996) density profile and constant rotational velocity  $v_\phi$ , Rahaman et al. (2014a) has demonstrated the possible existence of wormholes in the outer regions of the galactic halo. In another study, Rahaman et al. (2014b) have used the Universal Rotation Curve (URC) dark matter model and sample rotation curve utilizing an ansatz for  $v_\phi$  to obtain analogous results for the central parts of the halo. This letter is a significant sequel of the earlier results in a more refined manner and therefore confirms the possible existence of wormholes in most of the galaxies.

Basically by employing the experimental data of the rotational velocities of the outer regions of galaxies, we have estimated fifth degree polynomial that yields the expression for the velocity as a function of the radial coordinate  $r$ . The interpolation technique has been used to estimate this fifth degree polynomial. This rotational velocity is used to find the geometry of galactic halo

regions within the framework of general theory of relativity.

As a background of the present study we would like to report here briefly about the necessary density profiles in the following two paragraphs.

### 2.1 The Navarro-Frenk-White density profile

Navarro, Frenk & White (1996) proposed from the predictions of standard cold dark matter (CDM) cosmology N-body simulations, the structure of dark halos, in particular the density profile of dark halos. Their numerical simulations in the  $\Lambda$ -CDM scenarios led to the density profile of galaxies as

$$\rho(r) = \frac{\rho_s}{\left(\frac{r}{r_s}\right)\left(1 + \frac{r}{r_s}\right)^2} \quad (3)$$

where  $r_s$  is the characteristic scale radius and  $\rho_s$  the corresponding density. This density profile of CDM halos fits accurately up to masses between  $3 \times 10^{11} M_\odot$  –  $3 \times 10^{15} M_\odot$ .

### 2.2 The Universal Rotation Curve dark matter profile

According to the NFW model, the velocities in the central parts are too low (Gentile et al. 2004), but in the outer regions of the halo it fits well. In this case we consider with the range from closer to the center to the outer region where the URC dark matter profile (Castignani et al. 2012) is valid and given by

$$\rho(r) = \frac{\rho_0 r_0^3}{(r + r_0)(r^2 + r_0^2)}, \quad (4)$$

where  $r_0$  is the core radius and  $\rho_0$  is the effective core density.

The next ingredient is rotation curve regarding which it is believed that rotation curve analysis is one of the great support for the existence of dark matter in galaxies. By considering the  $H_\alpha$  data and also by adopting some radio rotation curves Persic et al. (1996) have analyzed a large number of rotation curves and have argued that rotation curves can be fitted not only for any luminosity, but also for any type of galaxies (may be spirals, low-surface-brightness ellipticals and dwarf-irregular galaxies). Therefore they used the term universal rotation curve (URC) in stead of rotation curves.

The tangential velocity (Chandrasekhar 1983) can be found from the flat rotation curve for the circular stable geodesic motion in the equatorial plane as

$$(v^\phi)^2 = r f'(r). \quad (5)$$

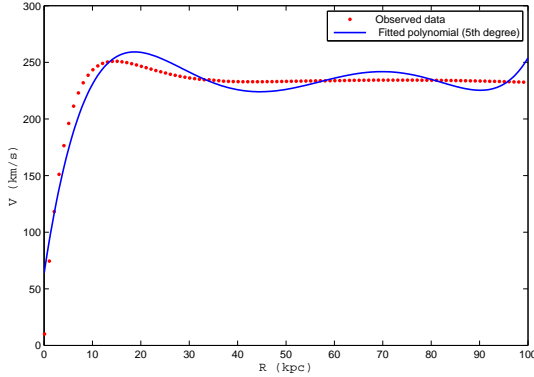
The velocity  $v^\phi$  in km/s of the rotation curve of the objects with total virial mass  $3 \times 10^{12}$  of solar masses in different radii  $r$  in kpc is given in Table 1 (Persic et al. 1996; Salucci et al. 2007). By using interpolation technique (Scarborough & Scarborough 1966) for the data given in Table 1, we have estimated fifth degree polynomial which is the best fitting curve that yields the expression for the velocity as a function of the radial coordinate  $r$ . Therefore the output can be presented as

$$v^\phi = 0.0000011r^5 - .0003r^4 + .031r^3 - 1.4r^2 + 28r + 64. \quad (6)$$

One can note that the velocity curve we obtained fits well with the observed tangential velocities in different distances (see Fig. 1).

Here, the velocity  $v^\phi$  is given in km/s which is equivalent to (Rahaman et al. 2008; Nandi 2009)

$$v^\phi = (0.000003)(0.0000011r^5 - .0003r^4 + .031r^3 - 1.4r^2 + 28r + 64). \quad (7)$$



**Fig. 1** The rotational velocity from observational data (Table 1) and estimated fifth degree polynomial. The horizontal and vertical lines are distance  $r$  in kpc and the rotation velocity  $v_\phi$  in km/sec respectively.

### 3 The Einstein field equations

Now, the Einstein field equations for the above metric (1) are

$$\frac{b'(r)}{r^2} = 8\pi\rho(r), \quad (8)$$

$$2\left(1 - \frac{b}{r}\right)\frac{f'}{r} - \frac{b}{r^3} = 8\pi p_r(r), \quad (9)$$

$$\left(1 - \frac{b}{r}\right)\left[f'' + \frac{f'}{r} + f'^2 - \left\{\frac{b'r - b}{2r(r-b)}\right\}\left(f' + \frac{1}{r}\right)\right] = 8\pi p_t(r). \quad (10)$$

From Eqs. (5) and (7), one can get the expression for redshift function  $f$  as

$$f(r) = (0.000003)^2[1.21 \times 10^{-13}r^{10} - 7.33 \times 10^{-11}r^9 + 1.96 \times 10^{-8}r^8 - 3.1 \times 10^{-6}r^7 + 3.1 \times 10^{-4}r^6 - 0.0207r^5 + 0.914r^4 - 24.81r^3 + 302.4r^2 + 3584r + 4096 \ln r]. \quad (11)$$

The solution of the same redshift function will be used to obtain the parameters from the Einstein field equations for different cases. The logic behind this is that this redshift function is obtained from the observed rotational velocities of the different distance of outer regions of galaxies.

We note that our solutions valid up from 0.1 kpc to 100 kpc. At the maximum distance of 100 kpc, the redshift function does not approach to zero. This means wormhole spacetime considered here is not asymptotically flat and therefore 100 kpc is the *cut off* radius where these solutions will be joined smoothly to an exterior vacuum solutions. Also we note that the redshift function is finite in the range  $0.1 \text{ kpc} < r < 100 \text{ kpc}$ . This finiteness of the redshift function actually prevents the event horizon.

Now in the following, we will discuss two cases separately: For the outer regions of the halo with  $30 \text{ kpc} \leq r \leq 100 \text{ kpc}$  (*Case 1*) whereas for the central region with  $9 \text{ kpc} \leq r \leq 30 \text{ kpc}$  (*Case 2*).

#### 3.1 Case 1

The Einstein field equation yields the shape function as

$$b(r) = 8\pi\rho_s r_s^3 \left[ \ln(r + r_s) + \frac{r_s}{r + r_s} \right] + C. \quad (12)$$

where  $C$  is an integration constant.

After obtaining both  $f(r)$  and  $b(r)$ , we now examine whether the spacetime possesses wormhole like geometry. From above, one can see that *redshift function* ( $f(r)$ ) remains finite to prevent an event horizon. We assume that characteristic scale radius  $r_s$  coincides with the throat radius of the wormhole, such that  $b(r_s) = r_s$ . This gives the value of the integration constant  $C$  as

$$C = r_s - 8\pi\rho_s r_s^3 \left[ \ln(2r_s) + \frac{1}{2} \right]. \quad (13)$$

Now, we find  $b'(r_s) < 1$ , to check the so-called *flare-out* condition. For the values  $\rho_s = 0.0000001$  and

$r_s = 30 \text{ kpc}$ , we obtain that  $b'(30) \approx 2.25 \times 10^{-4} < 1$ . Thus the flare-out condition holds good. One can note that this result does not modify if we implement the accurate values of Milky Way for  $\rho_s$  and  $r_s$  found out by Nesti & Salucci (1982).

The radial and lateral pressures assume the following forms:

$$p_r(r) = \frac{1}{4\pi} \left[ 1 - \frac{8\pi\rho_s r_s^3 (\ln(r+r_s) + \frac{r_s}{r+r_s} + C)}{r} \right] \times \left( \frac{f'}{r} - \frac{8\pi\rho_s r_s^3 (\ln(r+r_s) + \frac{r_s}{r+r_s} + C)}{8\pi r^3} \right), \quad (14)$$

$$p_t(r) = \frac{1}{8\pi} \left( 1 - \frac{8\pi\rho_s r_s^3 (\ln(r+r_s) + \frac{r_s}{r+r_s} + C)}{r} \right) \times \left[ f'' + \frac{f'}{r} + (f')^2 - \left\{ \frac{b'r - b}{2r(r-b)} \right\} \left( f' + \frac{1}{r} \right) \right], \quad (15)$$

where  $f(r)$  and  $b(r)$  are given in Eqs. (11) and (12) respectively and

$$f' = (0.000003)^2 [1.21 \times 10^{-12} r^8 - 6.6 \times 10^{-10} r^7 + 1.58 \times 10^{-7} r^6 - 2.168 \times 10^{-5} r^5 + 1.86 \times 10^{-3} r^4 - 0.1035 r^3 + 3.658 r^2 - 74.43 r + 604.8 + \frac{3584}{r} + \frac{4096}{r^2}], \quad (16)$$

$$f'' = (0.000003)^2 [10.8 \times 10^{-12} r^8 - 52.8 \times 10^{-10} r^7 + 11.1 \times 10^{-7} r^6 - 13.02 \times 10^{-5} r^5 + 9.3 \times 10^{-3} r^4 - 0.414 r^3 + 10.98 r^2 - 148.86 r + 604.8 - \frac{4096}{r^2}]. \quad (17)$$

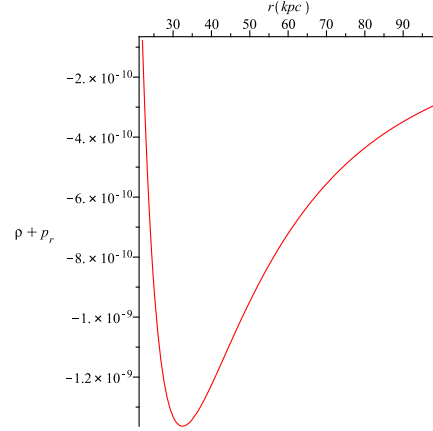
Fig. 2 indicates that  $(\rho + p_r) < 0$  and therefore the null energy condition is violated to hold a wormhole open.

### 3.2 Case 2

In this case the shape function can be found as

$$b(r) = 8\pi\rho_0 r_0^3 \left[ \frac{\log(r+r_0)}{2} + \frac{\log(r^2+r_0^2)}{4} - \frac{\tan^{-1}(\frac{r}{r_0})}{2} \right] + D, \quad (18)$$

where  $D$  is an integration constant.



**Fig. 2** The variation of the left-hand side of the expression for the null energy condition of matter in the galactic halo is plotted against  $r$ .

The radial and lateral pressures given by

$$p_r(r) = \frac{1}{4\pi} \left( 1 - \frac{8\pi\rho_0 r_0^3 \left( \frac{\log(r+r_0)}{2} + \frac{\log(r^2+r_0^2)}{4} - \frac{\tan^{-1}(\frac{r}{r_0})}{2} \right) + D}{r} \right) \times \left( \frac{f'}{r} - \frac{8\pi\rho_0 r_0^3 \left( \frac{\log(r+r_0)}{2} + \frac{\log(r^2+r_0^2)}{4} - \frac{\tan^{-1}(\frac{r}{r_0})}{2} \right) + D}{8\pi r^3} \right), \quad (19)$$

$$p_t(r) = \frac{1}{8\pi} \left[ 1 - \frac{8\pi\rho_0 r_0^3 \left( \frac{\log(r+r_0)}{2} + \frac{\log(r^2+r_0^2)}{4} - \frac{\tan^{-1}(\frac{r}{r_0})}{2} \right) + D}{r} \right] \times \left[ f'' + \frac{f'}{r} + f'^2 - \left\{ \frac{b'r - b}{2r(r-b)} \right\} \left( f' + \frac{1}{r} \right) \right], \quad (20)$$

where  $f(r)$  and  $b(r)$  are given in Eqs. (11) and (18).

One can check that the qualitative features meet all the requirements for the existence of a wormhole based on the URC model. Therefore, it would be desirable to examine the solutions critically. We assume that the throat of the wormhole coincides with the core radius  $r = r_0$ . As before,  $b(r_0) = r_0$  yields the value of the integration constant  $D$  where

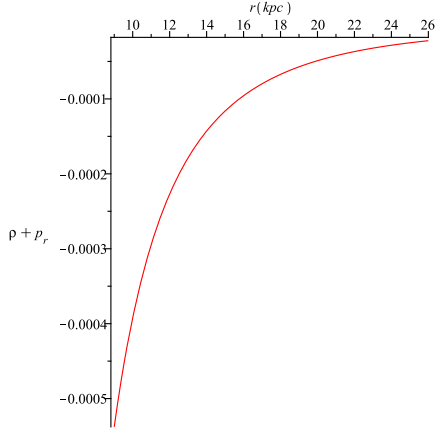
$$D = r_0 - 4\pi r_0^3 \rho_0 \ln 2r_0 - 2\pi r_0^3 \rho_0 \ln 2r_0^2 + \pi^2 \rho_0 r_0^3. \quad (21)$$

The known throat radius permits a closer look on the flare-out condition,  $b'(r_0) < 1$ . Using the observed values of the Milky Way galaxy,  $r_0 =$

9.11 kpc Maccio et al. (2012) and  $\rho_0 = 5 \times 10^{-24} (r_0/8.6 \text{ kpc})^{-1.1} \text{ g cm}^{-3}$  Castignani et al. (2012), we get

$$b'(r_0) \approx 1.74 \times 10^{-6} < 1.$$

Therefore, shape function obeys the flare-out condition. As before, Fig. 3 indicates that  $(\rho + p_r) < 0$ , therefore the null energy condition is violated to hold a wormhole open.



**Fig. 3** The variation of the left-hand side of the expression for the null energy condition of matter in the galactic halo is plotted against  $r$ .

## 4 Conclusion

Recently, based on Navarro-Frenk-White (NFW) (Navarro, Frenk & White 1996) density profile and the Universal Rotation Curve (URC) (Castignani et al. 2012) dark matter model, Rahaman et al. (2014a,b) have shown that the galactic halo possesses the necessary properties for supporting traversable wormholes. The former is valid for outer region of the galaxies whereas the latter is valid for the central parts of the galactic halo. For both the studies, they used the *ad hoc* functional forms of rotation velocity.

However, in this paper by using the experimental data of the rotational velocities at different distance of outer regions of galaxies, we have estimated fifth degree polynomial that yields the expression for the velocity as a function of the radial coordinate  $r$ . This rotational velocity is used to find the geometry of galactic halo regions within the framework of general theory of relativity. The estimated rotational velocity function  $v_\phi$  is well behaved within the range  $0.1 \text{ kpc} \leq r \leq 100 \text{ kpc}$ . We have used this rotational velocity function  $v_\phi$  to find the spacetime geometries of outer region as well as central parts of the galactic halo. Basically, we have in hand the data of galactic rotational velocities from

0.1 kpc to 100 kpc and we have estimated fifth degree polynomial which is the best fitted curve for velocity within this range. Therefore, from our model we cannot predict what happens in galactic centre. Also the wormhole exists outside the core of the galactic halo. Following Maccio et al. (2012) we have assumed core radius as 9.11 kpc. One can define the central region as 9.11 kpc up to 30 kpc and outer region as above 30 kpc. May be it needs further research to predict the event in the galactic centre.

In this respect it is to note that we do not know whether it is possible to explain the results without postulating the existence of wormholes, rather by attributing some specific physical properties to dark matter. However, according to our observations, the dark matter in the galactic halo region produces the space-time geometry which is very similar to wormhole like geometry.

The present study provides a clue for possible existence of wormholes in most of the galaxies and provides a theoretical platform to seek observational evidence for wormholes by studying the scattering of scalar waves or from past data using ordinary light as well as one can use the method of gravitational lensing as a possible experiment (Kuhfittig 2014). Another suggestion have given by Torres et al. (1998) that wormholes can be probed using light curves of gamma-ray bursts.

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**Table 1** The radius  $r$  in kpc and velocity  $v^\phi$  in km/s of the Rotation Curve of objects with total virial mass  $3 \times 10^{12}$  solar masses.

$R$ (kpc)	$v^\phi$ (km/s)	$R$ (kpc)	$v^\phi$ (km/s)	$R$ (kpc)	$v^\phi$ (km/s)
0.1	10.053	34.1	234.623	68.1	234.293
1.1	74.467	35.1	234.225	69.1	234.317
2.1	118.223	36.1	233.891	70.1	234.334
3.1	151.113	37.1	233.390	71.1	234.345
4.1	176.445	38.1	233.213	72.1	234.349
5.1	196.099	39.1	233.079	73.1	234.347
6.1	211.331	40.1	232.982	74.1	234.339
7.1	223.057	41.1	232.918	75.1	234.324
8.1	231.975	42.1	232.883	76.1	234.303
9.1	238.634	43.1	232.873	77.1	234.303
10.1	243.475	44.1	232.884	78.1	234.276
11.1	246.857	45.1	232.913	79.1	234.243
12.1	249.072	46.1	232.957	80.1	234.205
13.1	250.362	47.1	233.013	81.1	234.160
14.1	250.927	48.1	233.078	82.1	234.109
15.1	250.930	49.1	233.151	83.1	234.054
16.1	250.509	50.1	233.230	84.1	233.993
17.1	249.774	51.1	233.313	85.1	233.927
18.1	248.817	52.1	233.397	86.1	233.856
19.1	247.712	53.1	233.483	87.1	233.779
20.1	246.519	54.1	233.568	88.1	233.698
21.1	245.285	55.1	233.652	89.1	233.612
22.1	244.048	56.1	233.733	90.1	233.522
23.1	242.836	57.1	233.811	91.1	233.427
24.1	241.671	58.1	233.733	92.1	233.328
25.1	240.568	59.1	233.811	93.1	233.225
26.1	239.539	60.1	233.886	94.1	233.118
27.1	238.589	61.1	233.956	95.1	233.007
28.1	237.724	62.1	234.021	96.1	232.892
29.1	236.943	63.1	234.081	97.1	232.774
30.1	236.245	64.1	234.136	98.1	232.652
31.1	235.628	65.1	234.184	99.1	232.527
32.1	235.089	66.1	234.227		
33.1	234.623	67.1	234.263		